Microlocal and Global Analysis, Interactions with Geometry

ABSTRACTS

Organizers:

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MICROLOCAL AND GLOBAL ANALYSIS, INTERACTIONS WITH GEOMETRY

UNIVERSITY OF POTSDAM, FEBRUARY 21 – 25, 2022

Foreword

This conference which is part of a series of meetings in Potsdam initiated by Professor Schulze, is devoted to micro-local, singular and global analysis and their interactions with geometry and mathematical physics. It aims at enhancing interactions between micro-local and global analysis with geometry, reaching out to interactions with quantum field theory. The need to understand and describe the analysis and geometry of singular spaces has fostered connections and relations between a wide range of theories and approaches including scattering theory, pseudo-local analysis, index theory, foliations, Lorentzian geometry, spectral geometry, operator algebras .... and stimulates the use and development of a variety of techniques ranging from groupoids to Fourier integral operators.

This edition, like the 2021 edition, is a virtual meeting on Zoom due to the pandemic which impeds a meeting in person.

My warm thanks to Sylke Pfeiffer for her precious support with the organisation, to Chand Devchand for his valuable help in setting up the program and to David Prinz for his efficient work on this book of abstracts.

Sylvie Paycha
On behalf of the organising committee.

Speakers

Scientific Program					
Times	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Chair	*S. Paycha*	*V. Nistor*	*A. Strohmaier*	*W. Bauer*	*C. Bär*
9:00-9:40	**A. Strohmaier**	**G. Grubb**	**H. Chen**	**C. Bär**	**D. Tarama**
9:45–10:25	**B. Booss-Bavnbeck**	**L. Rodino**	**V. Nazaikinskii**	**A. Waters**	**Y. Wei**
10:30–10:50	Break				
Chair	*S. Paycha*	*V. Nistor*	*G. Grubb*	*W. Bauer*	*C. Bär*
10:50–11:20	**O. Fürst**	**G. Lee**	**A. Baldare**	**O. Islam**	**Z. Ruan**
11:25–12:05	**V. Nistor**	**E. Grong**	**P. Hintz**	**A. Savin**	**Y. Kordyukov**
12:15–12:55	**Y. Sanchez Sanchez / G. Vega-Molino**	**R. Darwich / G. Mattar Marriaga**	**U. Ludwig**	**V. Galkin**	**N. Popivanov**
13:00–13:45	Lunch Break		Disc. session w. lunch	Lunch Break	
Chair	*I. Witt*	*J. Seiler*	*J. Rowlett*	*U. Ludwig*	*S. Paycha*
13:45–14:15	**A. Laaroussi**	**P. McDonald**	Disc. session w. lunch	**O.L. Petersen**	**N. Savale**
14:20–15:00	**J. Rowlett**	**J. Toft**	**D.C. Sánchez / P. Schütte**	**C. Klein**	**T. Weich**
15:05–15:25	Break	D. Mazlum (book review)	Break		**Z. Yang**
15:25–16:05	**G. Mendoza**	**N. Teofanov**	**F. Baudoin**	**K. Yagdjian**	**M. Ruzhansky**
16:10–16:50	**E. Schrohe**	**A. Kutsenko**	**K. Bohlen**	**A. Galstyan**	
17:00–17:40	**S.I. Trapasso**				

Practical information:

- White talks are 40 minutes plus questions
- Yellow talks are 30 minutes plus questions
- Red talks are 20 minutes plus questions
- Discussion session with lunch: Please bring your lunch to the online discussion!
- All lectures will take place online in Zoom:
  Meeting-ID: 912 5022 0857
  Passcode: 529 518

Abstracts

Alexandre Baldare

Leibniz University Hannover 

TITLE: **Index of families of transversally elliptic operators with respect to central extensions.**

Consider a compact fibration $p : M \to B$ and let $\pi : P \to M$ be a G-principal bundle with G a compact Lie group. If $1 \longrightarrow \Gamma \longrightarrow \tilde{G} \xrightarrow{\zeta} G \longrightarrow 1$ is a central extension of G by an abelian finite group Γ then $p\circ\pi : P \to B$ becomes a $\tilde{G}$-fibration of compact manifolds with the action induced by ζ. Let $A = (A_b : C^\infty(P_b, E_b^+) \to C^\infty(P_b, E_b^-))_{b\in B}$ be a family of pseudodifferential operators. If the principal symbol of the family A is invertible on the vertical orthogonal to the orbit $T^*_{\tilde{G}}(P|B)$ then we can associate with A a distributional index with values in the de Rham cohomology of the base space B. In this talk, I will recall the definition of the index of families of transversally elliptic operators and its distributional index. I will then explain how to reduce the computation of the distributional index in the particular case of central extension and after that I will discuss the interesting example of families of Spin$(2n)$-transversally elliptic operators.

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Christian Bär

University of Potsdam 

TITLE: **Manifolds with many Rarita-Schwinger fields.**

The Rarita-Schwinger operator is the twisted Dirac operator restricted to 3/2-spinors. Rarita-Schwinger fields are solutions of this operator which are in addition divergence-free. This is an overdetermined problem and solutions are rare; it is even more unexpected for there to be large dimensional spaces of solutions. We prove the existence of a sequence of compact manifolds in any given dimension greater than or equal to 4 for which the dimension of the space of Rarita-Schwinger fields tends to infinity. These manifolds are either simply connected Kähler-Einstein spin with negative Einstein constant, or products of such spaces with flat tori. Moreover, we construct Calabi-Yau manifolds of even complex dimension with more linearly independent Rarita-Schwinger fields than flat tori of the same dimension. This is joint work with Rafe Mazzeo.

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Fabrice Baudoin

University of Connecticut 

TITLE: **Sobolev and L^p-Poincare inequalities in non-smooth spaces.**

We prove on some nested fractals scale invariant L^p-Poincaré inequalities on metric balls in the range $1 \leq p \leq 2$. Our proof is based on the development of the local L^p-theory of Korevaar-Schoen-Sobolev spaces using heat kernels

methods. Applications to scale invariant Sobolev inequalities and to the study of maximal functions and Hajłasz-Sobolev spaces on fractals are given. This is a joint work with Li Chen (Louisiana State University).

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Karsten Bohlen

University of Regensburg

TITLE: **Index theory for tensor products of pseudodifferential operators.**

In this talk we consider systems of tensor products of pseudodifferential operators. Motivating examples are double Cauchy integral operators that occur in the study of certain boundary value problems on bicylinders and the external tensor product of pseudodifferential operators that was first used in the work of M. F. Atiyah and I. Singer in 1968. We recall the definition of the so-called bisingular pseudodifferential calculus introduced by L. Rodino in 1975. The calculus is designed specifically to encompass all interesting examples of tensor products. Given a bisingular operator that is Fredholm, we discuss sufficient conditions for the Fredholm index to be expressible in terms of a topological index formula of the Atiyah-Singer type. The approach consists of the construction of an analogue of the "choose an operator" or Poincaré duality homomorphism, known in the standard case from the Baum-Douglas K-homology framework. To this end we define a suitable geometric model, in terms of a double deformation tangent groupoid, applicable to some types of tensor products.

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Bernhelm Booss-Bavnbek

Roskilde University

TITLE: **Counting the Number of 'Unwanted' Eigenvalues. Challenges and Solutions.**

In engineering design, negative eigenvalues of essentially positive operators can be a nuisance, e.g., due to resonance effects. There are various methods how to remove known eigenvalues by suitable small perturbations. Contrary to the procedures of linear algebra and to the treatment of linear systems of ordinary differential equations, in general there are no constructive methods of determining the eigenvalues of partial differential equations. Some time ago, the authors with collaborators (see also the work of Y. Latushkin and collaborators) have shown that the number of these "unwanted" eigenvalues can be identified with the Morse index and hence with the Maslov index of an associated curve of Fredholm Lagrangian subspaces in symplectic functional analysis, arising from a natural continuous variation of the original problem. That provided a constructive approach to determining the precise number of, e.g., all negative eigenvalues of a self-adjoint essentially positive operator with compact resolvent. These results, however, depend on technical preconditions regarding stronger or weaker versions of (A), the so-called weak inner Unique Continuation Property (wiUCP), and stronger or weaker versions of (B), the continuity of the Cauchy data spaces in all relevant Sobolev spaces, i.e., the continuous variation of the Calderón projection under smooth variation of the coefficients of the operator. In this talk we show how Property (B) can be derived in great generality solely by use of classical methods of the calculus of pseudo-differential operators.

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Hua Chen

Wuhan University 

TITLE: **Upper bound estimates of eigenvalues for Hörmander operators on non-equiregular sub-Riemannian manifolds.**

In this talk, we shall report on recent results on an eigenvalue problem for self-adjoint Hörmander operators on non-equiregular sub-Riemannian manifolds. Using the Rayleigh-Ritz formula and the sub-elliptic heat kernel estimates, we establish the upper bounds of eigenvalues which depend on the volume of subunit ball and the measure of the manifold. Under a certain condition, we obtain the explicit upper bounds of eigenvalues which have the polynomial growth in k with the optimal order related to the non-isotropic dimension of the manifold.

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Rayhana Darwich

University of Göttingen 

TITLE: **The propagation of polarization sets for first-order pseudodifferential systems with self-intersecting characteristics.**

Polarization sets were introduced by Dencker (1982) as a generalization of wave front sets to the vector-valued case. He also clarified the propagation of polarization sets when the characteristic variety of the pseudodifferential system under study consists of two hypersurfaces intersecting tangentially (1992). We generalize his result to the case of more than two intersecting characteristic hypersurfaces. We further indicate applications to the MHD equations.

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Oliver Fürst

Leibnitz University Hannover 

TITLE: **Index of Callias-type operators and generalized spectral shift functions.**

In this talk an approach to the index problem of Callias-type operators is given, taking the perspective of (relative) trace-class perturbation operator families. A Callias-type operator is the sum of a Dirac operator and the multiplication operator given by a family of self-adjoint operators acting in a Hilbert space H. The main goal of the talk is to establish a generalization of the Daletskii-Krein formula for derivations of operators and then construct an analogue of a Lifshits-Krein spectral shift function (SSF). Why are these objects related? F.Gesztesy and B.Simon showed that the Witten index, in some sense a generalization of the Fredholm index, and limits of the SSF are intimately related. W. Mueller noticed that in certain situations the value of the SSF of a pair of operators at 0 coincides with the spectral flow through zero of an operator family connecting the pair. Since M.Atiyah, V.Patodi, and I.Singer showed the "index=spectral flow"-theorem of operators of the form $\mathrm{d}/\mathrm{d}x + A(x)$, where $A(x)$ is a sufficiently regular operator family along the real line, it is suggestive that there is a version of "index=spectral flow" entirely formulated in terms of SSFs. Such a theorem was shown by A.Pushnitski, and furthermore he proved a generalized version of a trace formula connecting D and the endpoints of $A(x)$, due to C.Callias. The results of A. Pushnitski were generalized by F.Gesztesy, Y.Latushkin, K.Makarov, F.Sukochev, and Y.Tomilov to a case where $A(x)$ are relative trace-class perturbations, however still for a family parametrized over the real numbers, which is among other works by the aforementioned authors one of the most recent results in the field. The secondary objective

of this talk is to argue that the above constructed generalization of a SSF is the natural candidate for the right hand side of "index=spectral flow" in higher dimensions. This project is the continuation of my dissertation project under the supervision of M.Lesch, which was finished in early 2021.

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Valerii Galkin

Tyumen Industrial University & Surgut State University

TITLE: **The Space–Time Structures for the Smoluchowski–Boltzmann Kinetic Equation.**

Consider the mathematical model of the coagulation process in the disperse system that consists of a viscous fluid in which along the axis $\mathcal{O}x = \{x \in \mathbb{R}\}$ under the action of external forces move particles, which are associations of monomers, having a mass equals to $\mu_1 > 0$. This model is basic for the description of agglomeration processes in blood and oil moving in porous media. The main problem is to describe macroscopic geometric structures arising via local transfer of micro objects in the flow. The model of evolution for the concentration of the i-mers $f_i \overset{def}{=} f(i,x,t)$ in such a system based on the Smoluchowski–Boltzmann kinetic Equation

$$\frac{\partial f_i}{\partial t} + v_i \frac{\partial f_i}{\partial x} = \frac{1}{2}\sum_{j=1}^{i-1} \Phi_{i-j,j} f_{i-j} f_j - f_i \sum_{j=1}^{\infty} \Phi_{i,j} f_j, \quad i \in \mathbb{N}, \tag{1}$$

with initial data

$$f(i,x,0) = \varphi_i(x), \quad x \in \mathbb{R}, \quad i \in \mathbb{N}. \tag{2}$$

where $\Phi_{i,j} = \sigma_{i,j}|v_i - v_j|$ is given intensity of coagulation, which is symmetric, non-negative function on $\mathbb{N}\times\mathbb{N}$. The velocities of particles in the flow are v_i. Take the symbol $\bar{f} = T_t\varphi$ which determined by the formula $T_t(\varphi)(i,x) = \varphi_i(x - v_i t)$, $\varphi = \{\varphi_i\}_{i\in\mathbb{N}}$. Let us $\bar{f} = T_t\varphi$ and put $\bar{I}_i(x,t) = \sum_{j=1}^{\infty} \Phi_{i,j}\bar{f}_j(x,t)$.
THEOREM. *Let the capture cross-section of particles* $\sigma_{i,j}$ $(i,\ j \in \mathbb{N})$ *satisfies the inequalities* $0 < \inf\limits_{\mathbb{N}\times\mathbb{N}} \sigma_{i,j} \le \sup\limits_{\mathbb{N}\times\mathbb{N}} \sigma_{i,j} < \infty$, *and the velocity* v_i *is strictly monotone function on* $i \in \mathbb{N}$. *Assume that initial data in* (2) *are positive smooth functions on the set* $x \in \mathbb{R}$ *and* $\varphi_j(x) \to 0$ *when* $|x| \to \infty$. *Then at points where* $\bar{I}(x,t) = +\infty$, *the solution of the Cauchy problem* (1), (2) *has a singularity of the gradient catastrophe type and there arise spatially–time geometric structure.*

This research supported by RFBR grant 20-04-60123

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Anahit Galstyan

University of Texas Rio Grande Valley

TITLE: **The self-interacting Dirac fields in FLRW spacetime.**

In this talk we study solutions of the semilinear Dirac equation in the curved spacetime of the FLRW models of cosmology. We describe the relationship between the mass term, scale factor, nonlinear term, and initial function, which provides a global in time existence or an estimate on the lifespan of the solution of the Dirac equation in the expanding universe. The conditions on the imaginary part of mass will be discussed by proving nonexistence of the global solutions if certain relation between scale function and the mass are fulfilled.

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Erlend Grong

University of Bergen

TITLE: **Bounded curvature and functional inequalities for hypoelliptic operators.**

If we have a strongly elliptic second order differential operator L, then this operator corresponds to a Riemannian metric. There are several inequalities for the heat semi-group of L that is equivalent to a lower curvature bound for the metric. Due to recent work of Naber, Cheng and Thalmaier, there has now been found a similar collection of inequalities for the heat semi-group that are equivalent to a bounded curvature. The main tool to prove these functional inequalities is Malliavin calculus.

We want to establish a similar relationship between second order operators satisfying the strong Hörmander condition and bounded curvature in sub-Riemannian geometry. The results are from a joint work with Anton Thalmaier and Li-Juan Cheng.

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Gerd Grubb

University of Copenhagen

TITLE: **Boundary behavior of Dirichlet solutions to strongly elliptic fractional-order operators.**

The fractional Laplacian $(-\Delta)^a$, $0 < a < 1$, has been intensively studied in recent years, and much is known about the boundary behavior of the solutions to the homogeneous Dirichlet problem on domains in R^n. We shall here consider more general operators P, with x-independent symbol homogeneous of degree $2a$, but only assumed to be strongly elliptic. This leads to a new theory, where the transmission condition that was important for $(-\Delta)^a$, is replaced by a weaker condition, the *principal transmission condition*. However, the spaces where solutions belong, are still defined in terms of the so-called transmission spaces known from the treatment of $(-\Delta)^a$. The lecture will primarily be concerned with the model case, where the domain is a halfspace.

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Peter Hintz

ETH Zürich

TITLE: **Mode stability and shallow quasinormal modes of Kerr-de Sitter black holes.**

The Kerr-de Sitter metric describes a rotating black hole with mass m and specific angular momentum a in a universe, such as our own, with cosmological constant $\Lambda > 0$. I will explain a proof of mode stability for the scalar wave equation on Kerr-de Sitter spacetimes in the following setting: fixing Λ and the ratio $|a/m| < 1$ (related to the subextremality of the black hole in question), mode stability holds for sufficiently small black hole mass m. We also obtain estimates for the location of quasinormal modes (resonances) σ in any fixed half space $\Im\sigma > -C$. Our results imply that solutions of the wave equation decay exponentially in time to constants, with an explicit exponential rate. The proof is based on careful uniform estimates for the spectral family in the singular limit $m \to 0$ in which, depending on the scaling, the Kerr-de Sitter spacetime limits to a Kerr or the de Sitter spacetime.

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Onirban Islam

University of Leeds

TITLE: **A Gutzwiller trace formula for Dirac Operators on a stationary spacetime.**

The Duistermaat-Guillemin-Gutzwiller trace formula connects the spectral asymptotics of a differential operator with the underlying geometry. In this talk, I shall start with a few rather elementary identities as predecessors of this trace formula and then explain the Lorentzian framework by A. Strohmaier and S. Zelditch [Adv. Math. 376, 107434 (2021)]. The scalar setting is subsequently generalised for Dirac-type operators D on a spatially compact stationary spacetime. It has been found that the spectrum of the Lie derivative with respect to a Killing vector field, on $\ker(D)$ has discrete real eigenvalues and the trace of Cauchy evolution map has singularities at the periods of induced Killing flow on the space of lightlike geodesics. As a consequence of the singularity analysis, the Weyl law is achieved. (Based on arXiv:2109.09219 [math.AP]).

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Christian Klein

University of Burgundy

TITLE: **Large $|k|$ behavior of complex geometric optics solutions to d-bar problems.**

Complex geometric optics solutions to a system of d-bar equations appearing in the context of electrical impedance tomography and the scattering theory of the integrable Davey-Stewartson II equations are studied for large values of the spectral parameter k. For potentials $q \in \langle\cdot\rangle^{-2}H^s(\mathbb{C})$ for some $s \in]1,2]$, it is shown that the solution converges as the geometric series in $1/|k|^{s-1}$. For potentials q being the characteristic function of a strictly convex open set with smooth boundary, this still holds with $s = 3/2$ i.e., with $1/\sqrt{|k|}$ instead of $1/|k|^{s-1}$. The leading order contributions are computed explicitly. Numerical simulations show the applicability of the asymptotic formulae for the example of the characteristic function of the disk. This is work in collaboration with J. Sjöstrand and N. Stoilov.

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Yuri Kordyukov

Ufa Federal Research Center

TITLE: **Berezin-Toeplitz quantization on symplectic manifolds of bounded geometry.**

We will discuss the theory of Berezin-Toeplitz quantization on a class of noncompact symplectic manifolds, namely, on symplectic manifolds of bounded geometry. The quantum space is the spectral subspace of the renormalized Bochner Laplacian on a prequantum line bundle associated with some interval near zero. We show that this quantization has the correct semiclassical limit.

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Anton Kutsenko

Jacobs University Bremen

TITLE: **Fractal mountains and binary random loops.**

Each real number x from the interval $[-1,1]$ can be interpreted as an infinite random walk, where the steps correspond to binary digits of x. We consider functions $U(x)$ that count weighted loops in the random walk x. The graphs of such functions have a complex fractal structure reminiscent of mountain landscapes. We discuss the computation of various integrals of $U(x)$, Fourier transforms, connection with continued fractions and special functions, and interesting relations with Bernoulli polynomials, Pascal matrices, and determinants of Hessenberg matrices. Finally, we extended $U(x)$ to the multidimensional case $U(x_1,\dots,x_d)$ and discuss the possible applications of $U(x_1,\dots,x_d)$ to the hard open problems on the distribution of self-avoiding random walks that can be interpreted as zeros of U.

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Abdellah Laaroussi

Leibniz University Hannover

TITLE: **Heat kernel asymptotics for quaternionic contact manifolds.**

We consider the small time asymptotics for the heat kernel associated to the intrinsic sublaplacian on a quaternionic contact manifold considered as a subriemannian manifold. More precisely, we explicitly compute the first two coefficients appearing in the small time asymptotic expansion of the heat kernel on the diagonal. We show that the second coefficient equals the qc-scalar curvature (up to a constant multiple) associated to the canonical connection defined on such a manifold.

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Gihyun Lee

Max Planck Institute for Mathematics, Bonn

TITLE: **Magnetic Pseudodifferential Super Operators.**

In quantum mechanics, the time evolution of a state is determined by the Liouville equation $\frac{dho}{dt} = -\frac{i}{\hbar}(Hh_o - h_oH)$, where h_o is the density operator describing a given state and H is the Hamiltonian of a given system. Here we can observe that the Liouville operator $\mapsto L_Hh_o := -\frac{i}{\hbar}(Hh_o - h_oH)$ is a super operator, i.e., it assigns operators to operators. The resolvent $(L_H - \lambda)^{-1}$ of the Liouville operator L_H plays a central role in the study of quantum dynamics. Meanwhile, The main idea of various kinds of pseudodifferential calculi is to construct systematic ways of assigning operators to symbol functions, which enable us to translate properties of symbols to properties of associated pseudodifferential operators. In this talk, I will introduce a novel pseudodifferential calculus of super operators, and explain that the Liouville operator L_H and its resolvent $(L_H - \lambda)^{-1}$ can be incorporated into this new pseudodifferential theory. The asymptotic expansion of the product of symbols in small parameters will also be discussed. This talk is based on joint work with M. Lein.

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Oliver Lindblad Petersen

Uppsala University

TITLE: **Wave equations on expanding black hole spacetimes.**

I will present a proof that solutions to linear wave equations in a subextremal Kerr-de Sitter spacetime have asymptotic expansions in quasinormal modes up to a decay order given by the normally hyperbolic trapping. This extends earlier results of Vasy to the full subextremal range. The main novelties are a different way of obtaining a Fredholm setup that defines the quasinormal modes and a new analysis of the trapping of lightlike geodesics in the Kerr-de Sitter spacetime. In particular, this reduces the question of decay for solutions to wave equations to the question of mode stability. This is joint work with András Vasy.

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Ursula Ludwig

University of Duisburg-Essen & Max Planck Institute for Mathematics, Bonn

TITLE: **Bismut-Zhang theorem for singular spaces.**

The famous theorem of Cheeger and Müller states the equality between the analytic (or Ray-Singer) torsion and the topological torsion of a smooth compact manifold equipped with a unitary flat vector bundle. Using local index techniques and the Witten deformation Bismut and Zhang gave the most general comparison theorem of torsions for a smooth compact manifold. The aim of this talk is the generalisation of the Bismut-Zhang theorem to the context of isolated conical singularities: We first establish a comparison formula between the analytic torsion and a torsion, which we call the Bismut-Zhang torsion. We also establish anomaly formulas for all three terms in the comparison formula, i.e. we study how the terms behave under variations of the Riemannian conical metric.

Gisel Mattar Marriaga

University of Göttingen

TITLE: **Surface waves as Fourier distributions with complex phase.**

Surface waves are a special kind of solutions to certain hyperbolic BVPs which are, in particular, not uniformly stable. In elasticity, these waves are known as Rayleigh waves. By writing the solution operator as a Fourier integral operator with complex phase, we are able to further the analysis of this kind of singular solutions.

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Patrick McDonald

New College of Florida

TITLE: **Heat content and geometric analysis.**

The heat content associated to a bounded domain in a Riemannian manifold is a function obtained by solving an initial value problem for the heat operator on the domain. Heat content gives rise to a collection of geometric invariants closely related to the Dirichlet spectrum. In this talk I will survey recent results that compare and contrast the role of heat content invariants to the role of spectral data in geometric analysis. In particular, I will discuss results involving planar polygons and provide explicit examples of where heat content invariants and Dirichlet spectrum behave similarly, and also where they behave differently.

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Gerardo Mendoza

Temple University

TITLE: **Elliptic Sequences.**

I'll discuss some aspects of sequences of first order differential operators $P_q : C^\infty(M; E^q) \to C^\infty(M; E^{q+1})$ which are elliptic in the sense that the symbol sequence is exact at any nonzero covector of T^*M, but the the composition $P_{q+1} \circ P_q$ is not zero, so not a complex. Some features are similar to those of elliptic complexes: for instance, if M is compact, the sequence is finite, and the vector bundles E^q are Hermitian, then the index of the rolled up sequence is equal to the alternating sum of the dimensions of the null spaces of the Laplacians (this is not a deep result). But some are not, which make the problem interesting. Because of the exactness of the symbol sequence the operators $P_{q+1} \circ P_q$ are at most first order. I'll focus on the case where the order is in fact 0. Examples of sequences of the latter kind are those obtained with the aid of a connection, but these are not the only ones.

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Vladimir Nazaikinskii

Ishlinsky Institute for Problems in Mechanics, RAS

TITLE: **Pseudo-homogeneous Lagrangian manifolds and asymptotics in a Cauchy problem with localized initial data.**

We consider the Cauchy problem for the water wave equation $\eta_{tt} - |\nabla_x| \tanh(gD(x)|\nabla_x|)\eta = 0$ with localized initial data $\eta|_{t=0} = V(x/\mu)$, $\eta_t|_{t=0} = 0$, where μ is a small parameter characterizing the source size. The effective Hamiltonians $\lambda_\pm(x,p) = \pm\sqrt{|p| \tanh(gD(x)|p|)}$ of this problem have a singularity at $p = 0$ similar to that of a homogeneous function, and accordingly the Lagrangian manifolds used in the construction of the asymptotic solution of this problem have a singularity at $p = 0$. In the talk, we explain how to construct asymptotic solutions associated with this type of singularity of Lagrangian manifolds in this and other similar problems.

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Victor Nistor

University of Lorraine

TITLE: **Fredholm conditions for invariant pseudodifferential operators.**

Assume we have a group G acting on a bundle $E \to M$ and π an irreducible representation of G. We completely characterize the G-invariant pseudodifferential operators whose restriction to π-isotypical component is Fredholm. This is joint work with A. Baldare, R. Côme, and M. Lesch.

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Nedyu Popivanov

Bulgarian Academy of Sciences

TITLE: **Exponentially growing solutions to the (3+1)-D Protter-Morawetz problem.**

The Protter-Morawetz boundary value problem asks about solutions to the (3+1)-D wave equation outside a light cone with vertex on the plane $\{t = 0\}$, with no boundary conditions imposed at the light cone. The boundary conditions are given on the other part of the boundary: the retrograde wave cone and on $\{t = 0\}$. Over the years this problem has received considerable attention, amongst others by the namesakers and by Lax and Phillips. For classical solutions, it is overdetermined due to the fact that it has an infinite-dimensional cokernel. For a class of generalized solutions, however, one can solve this problem uniquely for many right-hand sides.

The following two facts are known (due to work by Schneider and myself):

(1) A generalized solution is bounded if and only if the right-hand side is orthogonal to the cokernel and to an infinite number of additional functions. In this case one has a classical solution.
(2) A generalized solution has polynomial growth at the vertex of the light cone if finitely many of these orthogonality conditions are violated.

The question remains what happens in case that both (1) and (2) do not hold. In this talk, we construct a smooth right-hand side such that the unique generalized solution exhibits exponential growth. This solution is found as a superposition of polynomially growing solutions. The latter are smooth up to the light cone, while being conormal with respect to the light cone when extended across the light cone to a solution of the wave equation. The order of conormality increases as more terms are added which eventually yields the strong exponential singularity. The constructions also show that the generalized solution so obtained cannot be extended across the light cone to a solution of the wave equation in the frame of the distribution theory.

The talk is based on joint work with Todor Popov (University of Sofia) and Ingo Witt (University of Göttingen).

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Luigi Rodino

University of Turin

TITLE: **Wigner microlocal analysis.**

We give a definition of wave front set in terms of Wigner transform. Action of pseudodifferential operators on it is studied. We note that the classical microregularity result for partial differential equations is false in this context,

because the presence of the ghost frequencies, having relevance in Quantum Mechanics. Applications are given to Schroedinger equations. Paper in collaboration with Elena Cordero, University of Torino.

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Julie Rowlett

Chalmers University & University of Gothenburg 

TITLE: **Trigonometric eigenfunctions, strictly tessellating polytopes, and crystallographic groups.**

This talk is based on joint work with my students, Max Blom, Henrik Nordell, Oliver Thim, and Jack Vahnberg. Trigonometric eigenfunctions are, as the name suggests, functions which can be expressed as a finite linear combination of sines and cosines. In 1980, Pierre Bérard proved that a certain type of polytopes in n dimensional Euclidean space, known as an alcoves, have trigonometric eigenfunctions for the Laplace eigenvalue equation with the Dirichlet boundary condition. In 2008, McCartin proved that in two dimensions, all such alcoves are of four types: rectangles, equilateral triangles, 30-60-90 triangles, and 45-45-90 triangles. In our work, we connect these results with the notion of 'strictly tessellating polytope.' We prove that the following are equivalent: the first Dirichlet eigenfunction for the Laplace eigenvalue equation on a polytope is real analytic on R^n; the polytope strictly tessellates R^n; the polytope is the fundamental domain of a crystallographic Coxeter group. Moreover, we prove that under any of these equivalent conditions, all of the Laplace eigenfunctions are trigonometric functions.

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Zhuoping Ruan

Nanjing University 

TITLE: **Elliptic boundary problems for edge-degenerate pseudodifferential operators.**

Let X be a compact $\mathscr{C}^\infty$ manifold with boundary, Y. A distribution u on $X \setminus Y$ is said to have a conormal asymptotic expansion of type P if, formally,

$$u(x,y) \sim \sum_{(p,k)\in P} \frac{(-1)^k}{k!}\, x^{-p} \log^k x\, u_{pk}(y) \quad \text{as } x \to +0,$$

where x is a boundary defining function and $u_{pk} \in \mathscr{C}^{-\infty}(Y)$. Given asymptotic types P, Q, we construct a calculus for edge-degenerate pseudodifferential operators A on $X \setminus Y$ with the property that if u has an asymptotic expansion of type P, then Au has an asymptotic expansion of type Q. The calculus contains the trace operators γ_{pk}, where $\gamma_{pk}u = u_{pk}$, while elliptic elements possess parametrices within the calculus. For $P = Q$ being the asymptotic type resulting from a Taylor series expansion at $x = 0$, we recover Boutet de Monvel's calculus.

This is joint work with Xiaochun Liu (Wuhan) and Ingo Witt (Göttingen).

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Michael Ruzhansky

Ghent University 

TITLE: **Subelliptic pseudo-differential operators on compact Lie groups.**

We will discuss here the pseudo-differential theory on compact Lie groups, adapted to subelliptic operators. The talk is based on the joint work with Duván Cardona.

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Duván Cardona Sánchez

Ghent University

TITLE: **Dixmier traces and Wodzicki residues on compact Lie groups: the paradigm of the global quantisation.**

In this talk we present a global analysis that we have developed for the explicit computation of the Dixmier trace and the Wodzicki residue of (elliptic and subelliptic) pseudo-differential operators on compact Lie groups. Our approach follows the paradigm of global quantisation instead of using the analysis under changes of coordinates. Joint work with Michael Ruzhansky and Julio Delgado.

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Yafet Sanchez Sanchez

Leibniz University Hannover

TITLE: **The Sobolev wavefront set of the causal propagator.**

In joint work with E. Schrohe, we showed that for non-smooth metrics the Sobolev wavefront set of the causal propagator is the same as in the smooth setting.

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Nikhil Savale

University of Cologne

TITLE: **Bergman-Szegö kernel asymptotics in weakly pseudoconvex finite type cases.**

Given a bounded domain $D \subset \mathbb{C}^n$ its Bergman kernel is the Schwartz kernel of the projector from the space $L^2(D)$ of square integrable functions on it, the space $H^0_{(0)}(D)$ of square integrable holomorphic ones. For a strongly-pseudoconvex domain (i.e. convex up to biholomorphism) the asymptotics of the Bergman kernel near the boundary were described in the classical work of Fefferman. This result was extended by Boutet de Monvel-Sjöstrand who gave a full parametrix for the Bergman kernel as a Fourier integral operator with complex phase. In this talk we give the natural extension of these results to weakly pseudoconvex domains in $\mathbb{C}^2$. An application to canonical metrics on the domain will be presented. Joint work with C.-Y. Hsiao.

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Anton Savin

RUDN University 

TITLE: **Eta-invariants of parameter-dependent families with periodic coefficients.**

We generalize the eta-invariant of Melrose to invertible elements in the operator algebra generated by parameter-dependent families of pseudodifferential operators on a closed smooth manifold and periodic functions of the parameter. This eta-invariant arizes in index problems for operators with shifts on manifolds with cylindrical ends. The talk is based on joint work in progress with K. Zhuikov (Moscow). The work is funded by RFBR and DFG, project number 21-51-12006.

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Elmar Schrohe

Leibniz University Hannover 

TITLE: **Equivariant Traces for an Algebra of Quantized Canonical Transformations on $\mathbb{R}^n$.**

We consider the algebra $\mathcal{B}$ of all operators on $L^2(\mathbb{R}^n)$ given as finite sums

$$B = \sum R_g T_w A,$$

where

- A is a zero order pseudodifferential operator in the Shubin calculus on $\mathbb{R}^n$
- For $w = a - ik \in \mathbb{C}^n$, T_w is the Heisenberg-Weyl operators given by $T_w u(x) = e^{ikx - iak/2} u(x-a)$, $u \in L^2(\mathbb{R}^n)$
- $g \mapsto R_g$ represents $g \in U(n)$ as a metaplectic operator, using the identification $\mathbb{C}^n \cong T^*\mathbb{R}^n$.

Choosing an auxiliary operator such as $H = |x|^2 - \Delta$, we obtain expansions for

- $\operatorname{Tr}(B(H-\lambda)^{-K})$ in powers of λ and $\log \lambda$ for K large, as $\lambda \to \infty$ in a sector of $\mathbb{C}$,
- $\operatorname{Tr}(Be^{-tH})$ as $t \to 0^+$ in powers of t and $\log t$, and
- the pole structure of the meromorphic extension of $\zeta_B(z) = \operatorname{Tr}(BH^{-z})$.

Moreover, we find a noncommutative residue that extends the Wodzicki residue to this situation and define equivariant traces on $\mathcal{B}$.

Joint work with Anton Savin, RUDN University, Moscow, Russia.

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Philipp Schütte

University of Paderborn 

TITLE: **Weighted Zeta Functions for Hyperbolic Flows.**

Pollicott-Ruelle resonances are discrete spectral invariants which have various applications in dynamical systems theory. They have been defined for hyperbolic systems such as geodesic flows on negatively curved manifolds, and in recent years their construction and investigation has mainly been driven by microlocal methods. One route towards Pollicott-Ruelle resonances is via the divisors of zeta functions defined in terms of the underlying dynamics. In this talk a new notion of weighted zeta function will be introduced and it will be explained how microlocal methods such as wave-front estimates enable us to prove their meromorphic continuation. The main building block of this proof is the meromorphically continued resolvent which relates to our zeta function via a weighted version of Guillemin's trace formula. This allows us to express the resonant states as residues of the weighed zeta function and provides a way of efficiently calculating the resonant states numerically in certain concrete models such as convex obstacle scattering and convex cocompact hyperbolic surfaces.

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Alexander Strohmaier

University of Leeds 

TITLE: **A spectral theory and microlocal approach to Casimir forces.**

I will define and discuss two spectral geometric local densities defined in terms of the functional calculus of the Laplace operator on an unbounded domain. These local quantities can for example be used to define the Casimir force and the Casimir energy of two objects, but they are also interesting in their own right. I will give an overview of recent results on the relation between the Casimir energy and the wave trace as well as spectral determinants of boundary layer operators. Joint work with T. Betcke, Xiaoshu Sun, Yan-Long Fang and Alden Waters.

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Daisuke Tarama

Ritsumeikan University 

TITLE: **Information Geometry and Dynamical Systems over Lie groups.**

This talk deals with a class of dynamical systems arising from information geometry of statistical models with Lie group symmetry. A statistical transformation model consists of a sample manifold, on which a Lie group smoothly acts, and a family of probability density functions invariant under the Lie group action. For a statistical transformation model, there are introduced a semi-Riemannian metric and a family of affine connections on the parameter Lie group, which are respectively called Fisher-Rao semi-definite metric and alpha-connections. In the talk, the geodesic flows induced from the Fisher-Rao semi-definite metric and the alpha-connections are considered in a special case where the Lie group is compact and semi-simple. The geometric formulation and the dynamical properties of the associated dynamical systems over Lie groups are analyzed.

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Nenad Teofanov

University of Novi Sad 

TITLE: **Subspaces of analytic functions and analytic pseudodifferential operators.**

As a modest contribution to the 200th anniversary of Fourier's "Théorie analytique de la chaleur" we first discuss Fourier transform invariant Gelfand–Shilov spaces, and Pilipović spaces. We also consider their Hermite series expansions. This point of view provides an extension of the relation $L^2(\mathbb{R}^d)$ and a space of analytic functions known as the Fock (or Bargmann) space. Namely, back in 1961. Valentine Bargmann published a paper in which he introduced an integral transform that now bears his name. His results provided a sound mathematical foundation to Vladimir Fock's study of canonical commutation relations of Schrödinger's approach to quantum mechanics. More recently (2012, 2017), Joachim Toft extended Bargmann's result for tempered distributions (from 1967) to various families of test functions and their distribution spaces. An outline of these fundamental results will be given in the first part of the talk. In the second part of the lecture we will present some recent results on analytic pseudodifferential operators obtained together with Joachim Toft and Patrik Wahlberg. In particular, we will consider Shubin type operators via the Bargmann kernel assignment. If the time permits, we will discuss a sharp Garding inequality in the context of

analytic pseudodifferential operators.

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Joachim Toft

Linnaeus University 

TITLE: **Analytic pseudo-differential calculus via the Bargmann transform.**

The Bargmann transform maps Fourier-invariant function spaces and their duals to spaces of formal power series expansions, which sometimes are convenient classes of analytic functions. In the 70th, Berezin used the Bargmann transform to translate problems in operator theory into an analytic pseudo-differential calculus, the so-called Wick calculus, where the involved symbols are analytic functions, and the corresponding operators map suitable classes of entire functions into other classes of entire functions. In the same manner, the Toeplitz operators correspond to so-called anti-Wick operators on the Bargmann transformed side. Recently, the author performed some investigations on the so-called Pilipovic spaces, defined by imposing suitable boundaries on the Hermite coefficients of the involved functions or distributions. The family of Pilipovic spaces contains all Fourier invariant classical Gelfand-Shilov spaces and other subspaces of such Gelfand-Shilov spaces. In the same way, the family of Pilipovic distribution spaces contains spaces which are strictly larger than any Fourier invariant Gelfand-Shilov distribution space (which in turn are superspaces to the set of tempered distributions). In the talk we find convenient characterisations of Wick and anti-Wick operators acting on the Bargmann images of Pilipovic spaces. We also discuss some links between global ellipticity in the real pseudo-differential calculus and the Wick calculus, as well as links between Wick and anti-Wick operators.

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S. Ivan Trapasso

University of Genova 

TITLE: **Phase space harmonic analysis of Feynman path integrals.**

The Feynman path integral formulation of quantum mechanics is universally recognized as a milestone of modern theoretical physics. Roughly speaking, the core principle of this picture states that the integral kernel of the Schrödinger evolution operator shall be expressed as a "sum over all possible histories of the system", namely as an integral on the infinite-dimensional space of suitable paths - to be interpreted in some sense as the limit of finite-dimensional short-time approximation operators. In spite of suggestive heuristic arguments and the success as a practical tool for performing computations, the quest for a mathematical theory of path integrals is far from over. This is evidenced by the wide variety of attempts to give rigorous meaning to this representation, mainly with the support of functional, harmonic and stochastic analysis. In particular, function spaces and techniques from phase space analysis have been fruitfully applied to the study of path integrals only in recent times. The purpose of this talk is to witness some results of this successful interplay. In particular, we will show how techniques of time-frequency analysis of pseudodifferential operators have made possible to advance in the research on mathematical path integrals with remarkable results.

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Gianmarco Vega-Molino

University of Bergen

TITLE: **An Horizontal Chern-Gauss-Bonnet formula on totally-geodesic foliations.**

We consider totally-geodesic foliations transverse to which exist a bracket-generating structure; under suitable conditions which arise from previous work on H-type foliations we show that the Euler characteristic can be computed purely from curvature invariants that are intrinsic to the transverse structure. The method of proof utilizes techniques involving the heat semigroup generated by the hypoelliptic sub-Laplacian on forms. This is a joint work with Fabrice Baudoin and Erlend Grong.

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Alden Waters

University of Groningen

TITLE: **The relative trace formula in electromagnetic scattering.**

Perfectly conducting obstacles in space change the scattering of electromagnetic fields. This talk establishes a relative trace formula for electromagnetic fields, effectively comparing the two scenarios using boundary layer theory. The trace formula is difficult to obtain due to the divergence free condition in Maxwell's equations in the absence of charge. I also discuss some of the necessary spaces for the Lipschitz case. This important formula relates the xi function to the scattering matrices for 1 forms.

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Yawei Wei

Nankai University

TITLE: **Properties of fractional p-Laplace equations with nonlinear and sign-changing potential.**

In this talk, we consider the fractional p-Laplace equations with nonlinear and sign changing potential. This work is in the inspiration of De Giorgi Conjecture. Here we explore the equivalence of maximum principle, strong maximum principle and the existence of the positive first eigenvalue. Then, we obtain that the solution is radially symmetric in the bounded domain, by applying the moving plane method. Finally, according to the idea of sliding method, we construct the proper auxiliary functions to conclude that the solution is monotone increasing in some direction in the unbounded domain. The different properties of the solutions in bounded and unbounded domain are mainly caused by the non-locality of the fractional p-Laplacian.

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Tobias Weich

University of Paderborn 

TITLE: **Microlocal methods in hyberbolic dynamics.**

Many geometrical objects come with a natural hyberbolic dynamical system. Examples are geodesic flows on manifolds of negative curvature or Weyl chamber flows on higher ranks locally symmetric spaces. In this talk I will review how microlocal analysis can be used to define a discrete spectrum for these dynamical sytems and how this leads to several important applications such as the continuation of dynamical zeta functions of the construction of invariant measures.

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Karen Yagdjian

University of Texas Rio Grande Valley 

TITLE: **Huygens' principle for the fields in spacetime of non-constant curvature.**

In this talk we will discuss sufficient and necessary conditions for the validity of the Huygens' principle for the Dirac and Klein-Gordon equations in the non-constant curvature spacetime of the expanding universe. The Huygens' principle will be considered for the equations of fields undergoing a red shift of their wavelengths as the universe expands.

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Zhipeng Yang

University of Göttingen 

TITLE: **Harmonic analysis on 2-step nilpotent Lie groups.**

I will present some basic analysis results on 2-step nilpotent Lie groups $\mathbb{G}$. In particular, based on explicit knowledge of the irreducible unitary representation of $\mathbb{G}$, I will perform a Fourier analysis of the sub-Laplacian and give a closed formula for the heat kernel of the sub-Laplacian on $\mathbb{G}$.

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